Spatial exponential decay of the ground state of the renormalized Nelson model by Feynman-Kac formula

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This is a joint work with Oliver Matte. The Nelson Hamiltonian

$$H_{\hat{\varphi}} = -\frac{1}{2}\Delta + V + H_f + H_I(\hat{\varphi})$$

is a scalar model in quantum field theory, which ultraviolet cutoff $\hat{\varphi}$ is renormalized, and the existence of the renormalized Hamiltonian H_∞ is known. H_∞ is a self-adjoint operator in $L^2(\mathbb{R}^3) \otimes \mathcal{F}$. Spectrum of H_∞ has been studied, and in particular, the existence and uniqueness of the ground state Φ of H_∞ were proven. In this talk, we give a rigorous evaluation of the spatial decay of the ground state. From a mathematical point of view, the difficulty of the discussion lies in the fact that the specific form of H_∞ cannot be specified. For this reason, we present FKF of the semigroup generated by H_∞. Then the result is

$$ae^{-b\int_0^T \sqrt{2V_{sup}(\gamma(s))}|\dot{\gamma}(s)|ds} \leq \|\Phi(x)\|_{\mathcal{F}} \leq ce^{-d|x|\sqrt{V_{inf}(x)+\delta}}.$$

Here a, b, c, d and δ are constants and $\gamma \in C^1([0,T] : \mathbb{R}^3)$ with $\gamma(0) = 0$ and $\gamma(T) = x$. In particular suppose that $\alpha|x|^{2n} \leq V(x) \leq \beta|x|^{2n}$ for large $|x|$. Then

$$a'e^{-b'|x|^{n+1}} \leq \|\Phi(x)\|_{\mathcal{F}} \leq c'e^{-d'|x|^{n+1}}.$$